**SYLLABUS**

**Fall semester 2020-2021 academic years**

**on the educational program of the course “\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_”**

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| **Discipline’s code** | **Discipline’s title** | **Independent work of students (IWS)** | **No. of hours per week** | | | | | **Number of credits** | **Independent work of student with teacher (IWST)** |
| **Lectures (L)** | **Practical training (PT)** | | **Laboratory (Lab)** | |
|  | Variation calculus and optimization methods |  | 2 | 1 | | 0 | | 3 |  |
| **Academic course information** | | | | | | | | | |
| **Form of education** | **Type of course** | **Types of lectures** | | | **Types of practical training** | | **Number of IWS** | | **Form of final control** |
|  |  |  | | |  | |  | |  |
| Lecturer | S. Ya. Serovajsky, doctor of science, professor | | | | | |  | | |
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| **Academic presentation of the course** |

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| **Aim of course** | **Expected Learning Outcomes (LO)**  As a result of studying the discipline the undergraduate will be able to: | **Indicators of LO achievement (ID)**  (for each LO at least 2 indicators) |
| The main purpose of the course is to familiarize students with the fundamental concepts of calculus of variations and optimization control theory | LO1 Introduction | ID1.1 Applications of extremum theory  ID1.2 Function minimization theory |
| LO2 Variation calculus | ID2.1 Euler equation for Lagrange problem.  ID2.2 Lagrange problem for the function family.  ID2.3 Lagrange problem with high derivatives.  ID2.4 Lagrange Problem for functions with many variables.  ID2.5 Bolza Problem.  ID2.6 Variational problems with isoperimetric conditions.  ID2.7 Variational problems with pointwise constraints. |
| LO3 Optimization methods | ID3.1 Easiest optimization control problems.  ID3.2 Optimization control problems for the vector case.  ID3.3 Optimization control problem with fixed final state. |
| LO4 Addition | ID4.1 Differentiation of functionals and abstract optimization problems.  ID4.2 Variational inequalities.  ID4.3 Existence and uniqueness of extremum problems.  ID4.4 Inverse problems |
| **Prerequisites** | Mathematical analysis, differential equations | |
| **Post requisites** | Special courses | |
| **Information resources** | 1. Васильев Ф.П. Методы оптимизации. В двух томах. – М.: МЦНМО, 2011. 2. Serovajsky S. Practical Course of the Optimal Control Theory with Examples. Almaty, Қазақ университеті, 2011. 3. Kirk D. E. Optimal Control Theory: An Introduction. – New Jersey, Englewood Cliffs, 2004. 4. Будылин А.М. Вариационное исчисление. – Санкт-Петербург, СПбГУ, 2001. 5. Лутманов С.В. Курс лекций по методам оптимизации. – Ижевск, 2001. 6. Алексеев В. М., Тихомиров В. М., Фомин С. В. Оптимальное управление. – М., Наука, 2000. 7. Эльсгольц Л.Э. Дифференциальные уравнения и вариационное исчисление. – М., Наука, 2000. 8. [http://www.newlibrary.ru/book/budylin\_a\_m\_/variacionnoe\_ischislenie.html](http://www.newlibrary.ru/book/budylin_a_m_/variacionnoe_ischislenie.html%20) . | |

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| **Academic policy of the course in the context of university moral and ethical values** | **Academic Behavior Rules:**  All students have to register at the MOOC. The deadlines for completing the modules of the online course must be strictly observed in accordance with the discipline study schedule.  ATTENTION! Non-compliance with deadlines leads to loss of points! The deadline of each task is indicated in the calendar (schedule) of implementation of the content of the curriculum, as well as in the MOOC.  **Academic values:**  - Practical trainings/laboratories, IWS should be independent, creative.  - Plagiarism, forgery, cheating at all stages of control are unacceptable.  - Students with disabilities can receive counseling at e-mail \*\*\*\*\*\*\*@gmail.com. |
| **Evaluation and attestation policy** | **Criteria-based evaluation:**  assessment of learning outcomes in relation to descriptors (verification of the formation of competencies in midterm control and exams).  **Summative evaluation:** assessment of work activity in an audience (at a webinar); assessment of the completed task. |

**CALENDAR (SCHEDULE) THE IMPLEMENTATION OF THE COURSE CONTENT:**

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| --- | --- | --- | --- | --- | --- | --- | --- |
| weeks | Topic name | LO | ID | amount of hours | Maximum score | Form of Knowledge Assessment | The  Form of the lesson  / platform |

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| **Module I. Introduction** | | | | | | | |
| 1 | **L 1 Practical examples of the extremum problems. Minimization of functions.** | LО 1 | ID 1.1,1.2 | 2 | 5 |  | Video lecture  in MS Teams |
| 1 | **PT 1** Function minimization theory | LО 1 | ID 1.2 | 1 | 15 |  | Webinar  in MS Teams |
| **Module П Variation calculus** | | | | | | | |
| 2 | **L 2 Euler equation for Lagrange problem.** Lagrange problem. Euler equation. Examples. The fall of the body. Fermat principle and the refraction of light law. | LO 2 | ID 2.1 | 2 | 5 |  | Video lecture  in MS Teams |
| 2 | **PT 2** Euler equation for Lagrange problem. | LO 2 | ID 2.1 | 1 | 15 |  | Webinar  in MS Teams |
| 3 | **L 3 Lagrange problem for the function family.** Problem statement. The system of Euler equations. Example. Principle of the least action | LO 2 | ID 2.2 | 2 | 5 |  | Video lecture  in MS Teams |
| 3 | **PT 3** Lagrange problem for the function family. | LO 2 | ID 2.2 | 1 | 15 |  | Webinar  in MS Teams |
| 4 | **L 4 Lagrange problem with high derivatives.** Problem statement. Euler – Poisson Equation. Example. Bending of the elastic beam. | LО 2 | ID 2.3. | 2 | 5 |  | Video lecture  in MS Teams |
| 4 | **PT 4** Lagrange problem with high derivatives. | LО 2 | ID 2.3. | 1 | 15 |  | Webinar  in MS Teams |
| 5 | **L 5 Lagrange Problem for functions with many variables.** Problem statement. Ostrogradsky equation. Dirichlet integral. The oscillation of the string. | LО 2 | ID 2.4. | 2 | 5 |  | Video lecture  in MS Teams |
| 5 | **PT 5** Lagrange Problem for functions with many variables. | LО 2 | ID 2.4. | 1 | 15 |  | Webinar  in MS Teams |
| 5 | **MT 1** | LО 1-2 |  |  | 100 |  |  |
| 6 | **L 6 Bolza Problem.** Problem statement. Necessary conditions of extremum. Transversality conditions. Example. River crossing problem. | LО 2 | ID 2.5. | 1 | 5 |  | Video lecture  in MS Teams |
| 6 | **PT 6** Bolza Problem | LО 2 | ID 2.5. | 2 | 15 |  | Webinar  in MS Teams |
| 7 | **L 7 Variational problems with isoperimetric conditions.** Problems with isoperimetric condition. Lagrange multipliers method. A spectrum problem. The problem with many isoperimetric conditions | LО 2 | ID 2.6. | 1 | 5 |  | Video lecture  in MS Teams |
| 7 | **PT 7** Variational problems with isoperimetric conditions. | LО 2 | ID 2.6. | 2 | 15 |  | Webinar  in MS Teams |
| 8 | **L 8 Variational problems with pointwise constraints.** Problem statement. Lagrange multipliers method. Example. Oscillation of the pendulum. | LО 2 | ID 2.7. | 1 | 5 |  | Video lecture  in MS Teams |
| 8 | **PT 8** Variational problems with pointwise constraints. | LО 2 | ID 2.7. | 2 | 15 |  | Webinar  in MS Teams |
| **Module IП Optimization methods** | | | | | | | |
| 9 | **Easiest optimization control problems.** Maximization of the flight of the missile (problem statement). Pontyagin’s maximum principle. Example. Iterative method for solving the optimality conditions. | LО 3 | ID 3.1. | 2 | 5 |  | Video lecture  in MS Teams |
| 9 | **PT 9** Easiest optimization control problems. | LО 3 | ID 3.1. | 1 | 15 |  | Webinar  in MS Teams |
| 10 | **L 10 Optimization control problems for the vector case.** Problem statement. Pontyagin’s maximum principle. Example. Maximization of the flight of the missile (solving). | LО 3 | ID 3.2. | 2 | 5 |  | Video lecture  in MS Teams |
| 10 | **PT 10** Optimization control problems for the vector case. | LО 3 | ID 3.2. | 1 | 15 |  | Webinar  in MS Teams |
| 10 | **МТ (Midterm Exam)** | LО 2-3 |  |  | 100 |  |  |
| 11 | **L 11 Optimization control problem with fixed final state.** Problem Statement.Maximum principle. Example. Time optimization problem. Firing method. | LО 3 | ID 3.3. | 2 | 5 |  | Video lecture  in MS Teams |
| 11 | **PT 11** Optimization control problem with fixed final state | LО 3 | ID 3.3. | 1 | 15 |  | Webinar  in MS Teams |
| **Module IV Addition** | | | | | | | |
| 12 | **L 12 Differentiation of functionals and abstract optimization problems.** Gradient methods for functions. Gateau derivatives of functionals. Examples. Gradient methods for functionals | LО 4 | ID 4.1. | 2 | 5 |  | Video lecture  in MS Teams |
| 12 | **PT 12** Differentiation of functionals and abstract optimization problems | LО 4 | ID 4.1. | 1 | 15 |  | Webinar  in MS Teams |
| 13 | **L 13 Variational inequalities**. Variational inequalities and constraints minimization of functional. Examples. Variational inequalities and constraints minimization of functional | LО 4 | ID 4.2. | 2 | 5 |  |  |
| 13 | **PT 13** Variational inequalities | LО 4 | ID 4.2. | 1 | 15 |  | Video lecture  in MS Teams |
| 14 | **L 14**. **Existence and uniqueness of extremum problems.** Existence theorem for abstract optimization problems. Uniqueness theorem for abstract optimization problems. Example. | LО 4 | ID 4.3 | 2 | 5 |  | Video lecture  in MS Teams |
| 14 | **PT 14** Finite difference method | LО 4 | ID 4.3. | 1 | 15 |  | Webinar  in MS Teams |
| 15 | **L 15 Inverse problems of mathematical physics.** Identification of the mathematical models. Direct and inverse problems of mathematical physics. Inverse problems of mathematical physics and the theory of extremum. | LО 4 | ID 4.4. | 1 | 5 |  | Video lecture  in MS Teams |
|  | **PT 15** Inverse problems | LО 4 | ID 4.4. | 2 | 15 |  | Webinar  in MS Teams |
|  | **MT 2** | LО 4-6 |  |  | 100 |  |  |

[Abbreviations: QS - questions for self-examination; TK - typical tasks; IT - individual tasks; CW - control work; MT - midterm.

Comments:

- Form of L and PT: webinar in MS Teams / Zoom (presentation of video materials for 10-15 minutes, then its discussion / consolidation in the form of a discussion / problem solving / ...)

- Form of carrying out the CW: webinar (at the end of the course, the students pass screenshots of the work to the monitor, he/she sends them to the teacher) / test in the Moodle DLS.

- All course materials (L, QS, TK, IT, etc.) see here (see Literature and Resources, p. 6).

- Tasks for the next week open after each deadline.

- CW assignments are given by the teacher at the beginning of the webinar.]

**Dean**

**Chairman of the Faculty Methodical Bureau**

**Head of the Department**

**Lecturer**